Assignment 1

Due August 12, 2019

1. Solve the following system of linear equations:

$$\begin{aligned} x - 3z &= -2 \\ -3x + y + 6z &= 3 \\ 2x - 2y - z &= -1 \end{aligned}$$

2. Solve the following system of linear equations:

$$\begin{aligned} -x_1 + 2x_2 - x_3 &= 2 \\ -2x_1 + 2x_2 + x_3 &= 4 \\ 3x_1 + 2x_2 + 2x_3 &= 5 \\ -3x_1 + 8x_2 + 5x_3 &= 17 \end{aligned}$$

3. Suppose $AB = \begin{bmatrix} 5 & 4 \\ -2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}$. Find A.

4. Compute the Hessian matrix for the following functions:
   (a) $f(x, y) = 4x^2y - 3xy^3 + 6x$
   (b) $f(x, y) = 3x^2y - 7x\sqrt{y}$

5. Calculate the determinant for the following matrix:

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 4 & 2 \\ 1 & 4 & 3 \end{pmatrix}$$

6. Let X be an $n \times n$ matrix. Show $X^{-1} = \left(X^T X\right)^{-1} X^T$

7. Determine the definiteness of the following matrix:

$$\begin{pmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

8. Find the critical points and classify these as local max, local min, saddle point, or "can't tell":

$$f(x, y, z) = \left(x^2 + 2y^2 + 3z^2\right) e^{-\left(x^2+y^2+z^2\right)}$$

9. The Cobb-Douglas utility function is given by $U(x_1, x_2) = kx_1^{\alpha}x_2^{1-\alpha}$ where x_1 and x_2 are two goods. Assume a consumer's budget set is $p_1x_1 + p_2x_2 \leq I$. Do the following:

(a) List the Karush Kuhn Tucker conditions for the problem above.

(b) Solve for the maximizers x_1^*, x_2^*, and the Lagrangian multiplier.

10. Consider the following production function: $y = f(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} x_i^{\alpha_i}$ for $i = 1, 2, \ldots, n$ where $a_i > 0$ and $\sum_{i=1}^{n} \alpha_i = 1$. The firm maximizes profits under perfect competition (in other words price or output, $p > 0$, and prices of inputs, $w_i > 0$, are exogenous or given):

$$\max_{x_1, x_2, \ldots, x_n} pf(x_1, x_2, \ldots, x_n) - \sum_{i=1}^{n} w_i x_i$$

Solve for the maximizer $(x_1^*, x_2^*, \ldots, x_n^*)$

11. Use integration by parts to evaluate the following integral:

$$\int x^2 sin(x) dx$$

12. Use the chain rule to find $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial s}$ for

$$w = 2xy$$

where $x = s^2 + t^2$ and $y = \frac{s}{t}$.