Master's Mathcamp Final
Summer 2019
Time Limit: 60 Minutes

Name: ______________________

Instructions: Some questions on this test may be a bit difficult. Relax, and answer all questions to the best of your ability (check every page to make sure you have answered everything). Note that partial solutions will receive partial credit, so putting something for a question will be better than leaving that question blank.

Here are some useful derivatives:

$$\frac{d\ln x}{dx} = \frac{1}{x}$$

$$\frac{d\ln f(x)}{dx} = \frac{f'(x)}{f(x)}$$

$$\frac{dx^n}{dx} = nx^{n-1}$$

$$\frac{df(x)^n}{dx} = nf'(x)f(x)^{n-1}$$

1. **(40 points)** Consider the following consumer utility max problem (and assume price $m \geq p \geq 1$):

$$\max_{x_1,x_2} \ln x_1 + x_2$$

$$\text{such that}$$

$$x_1 + px_2 \leq m$$

   (a) Calculate the Hessian, $D^2 f_{(x_1,x_2)}$, of the objective function, $f(x_1, x_2) = \ln x_1 + x_2$, and show that it is negative semi-definite over the domain $x_1 \geq 0, x_2 \geq 0$ (and thus f is concave).

(b) Define the Lagrangian and find the Karush-Kuhn-Tucker conditions (you don't have to include the nonnegativity constraints).

(c) The objection function is "strictly increasing" in both inputs (x_1 and x_2) meaning that either increasing x_1 or x_2 will increase one's utility. That means that the budget constraint ($x_1 + px_2 \leq m$) is binding, or in other words holds with equality ($x_1 + px_2 = m$). Give economic intuition why this is the case.

(d) Using the conditions in (b) and (c), find the maximizers, x_1^* and x_2^*.

2. **(15 points)** Use integration by parts to evaluate the following integral:

$$\int x\sqrt{x+1}\,dx$$

3. **(15 points)** Solve the following system of linear equations:

$$\begin{aligned} 2x + 2y - z &= 2 \\ x + y + z &= -2 \\ 2x - 4y + 3z &= 0 \end{aligned}$$