WSU Economics PhD Mathcamp Notes

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July 26, 2019

These notes are to accompany Mathematics for Economists by Simon and Blume.

1 Logic

1.1 Statements

A **statement** is a declarative sentence or assertion that is either true or false. They are often labelled with a capital letter (P, Q, R are most commonly used). Below are examples of statements:

P_1: The integer 6 is even.
P_2: A square has 5 sides.

Notice that the first statement is true, whereas the second statement is false.

1.2 Open Sentences

An **open sentence** is similar to a statement, except it contains one or more variables. Below are examples of open sentences:

P_3: The integer k is even.
P_4: A square has j sides.

Notice that in the first open sentence, there exists many values of k where the statement holds true. In the second open sentence, the statement holds true only if $j = 4$.

1.3 Negation

The **negation** of a statement (or proposition) P is denoted by $\sim P$ or $\neg P$, and is pronounced "not P". $\sim P$ is the opposite of P. The example below shows a statement P_5, and its negation $\sim P_5$:

P_5: The integer 7 is odd.
$\sim P_5$: The integer 7 is even.

Recall that a statement or open sentence can only take on one of two values: true or false. Thus, the negation of a statement or open sentence will take on the opposite truth value. This can be seen in the following truth table:

P	$\sim P$
T	F
F	T

Table 1: Truth table for P and $\sim P$.

2 Logical Connectives

2.1 Disjunction

The **disjunction** of the statements P and Q is denoted as $P \vee Q$ is defined as the statement P or Q. $P \vee Q$ is true if either P or Q is true, otherwise it is false. Notice that from the first example, $P_1 \vee P_2$ is true since P_1 is true and P_2 is false. Below is a truth table for $P \vee Q$.

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Table 2: Truth table for $P \vee Q$.

2.2 Conjunction

The **conjunction** of the statements P and Q is denoted as $P \wedge Q$ is defined as the statement P and Q. $P \wedge Q$ is true if either P and Q are both true, otherwise it is false. Notice that from the first example, $P_1 \wedge P_2$ is false since P_1 is true and P_2 is false. Below is a truth table for $P \wedge Q$.

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Table 3: Truth table for $P \wedge Q$.

2.3 Implication and Biconditional

An **implication** is usually denoted as $P \Rightarrow Q$, and means either "If P, then Q" or "P implies Q". Below is a truth table for $P \Rightarrow Q$.
There are multiple ways of expressing $P \Rightarrow Q$:

P implies Q
If P, then Q
P only if Q
P is sufficient for Q
Q if P
Q is necessary for P

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Table 4: Truth table for $P \Rightarrow Q$.

$Q \Rightarrow P$ is called the **converse** of $P \Rightarrow Q$. If $P \Rightarrow Q$ is true, it's not necessarily the case that its converse, $Q \Rightarrow P$, is true.

A **biconditional** of P and Q is usually denoted by $P \Leftrightarrow Q$, and means $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$. There are multiple ways of expressing $P \Rightarrow Q$:

$$P \text{ if and only if } Q$$
$$P \text{ iff } Q$$
$$P \text{ is equivalent to } Q$$

Below is a truth table for $P \Leftrightarrow Q$:

P	Q	$P \Rightarrow Q$	$P \Leftarrow Q$	$P \Leftrightarrow Q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Table 5: Truth table for $P \Leftrightarrow Q$.

2.4 Compound Statements

The operators explained before ($\sim, \vee, \wedge, \Rightarrow, \Leftrightarrow,$ and $\Leftrightarrow$) are referred to as logical connectors. The combination of at least one statement and at least one connector is called a **compound statement**. Notice that the following are compound statements:

$$\sim P$$
$$P \vee Q$$
$$P \Rightarrow Q$$
$$(P \wedge Q) \wedge (Q \Rightarrow \sim P)$$

2.5 Tautologies

A compound statement is a **tautology** if all possible truth values are true. An example of a tautology is $P \vee (\sim P)$. The following truth table shows that all the possible truth values are true:

2.6 Contradictions

A compound statement is a **contradiction** if all possible truth values are false. An example of a contradiction is $P \wedge (\sim P)$. The following truth table shows that all the possible truth values are true:

P	$\sim P$	$P \vee (\sim P)$
T	F	T
F	T	T

Table 6: Truth table for $P \vee (\sim P)$.

P	$\sim P$	$P \wedge (\sim P)$
T	F	F
F	T	F

Table 7: Truth table for $P \wedge (\sim P)$.

2.7 Logical Equivalence

Two compound statements R and S are **logically equivalent** if they have the same truth values in a truth table. If R and S are logically equivalent, then we write $R \equiv S$. For example, we see that $P \implies Q$ and $(\sim P) \vee Q$ are logically equivalent as all truth values for $P \implies Q$ and $(\sim P) \vee Q$ are the same:

P	Q	$P \Rightarrow Q$	$\sim P$	$(\sim P) \vee Q$
T	T	**T**	F	**T**
T	F	**F**	F	**F**
F	T	**T**	T	**T**
F	F	**T**	T	**T**

Table 8: Truth table for $P \Leftrightarrow Q$.

2.8 De Morgan's Laws

De Morgan's Laws are defined as:

1. $\sim (P \vee Q) \equiv (\sim P) \wedge (\sim Q)$
2. $\sim (P \wedge Q) \equiv (\sim P) \vee (\sim Q)$

Let's show that $\sim (P \vee Q)$ and $(\sim P) \wedge (\sim Q)$ are logically equivalent.

P	Q	$P \vee Q$	$\sim P$	$\sim Q$	$\sim (P \vee Q)$	$(\sim P) \wedge (\sim Q)$
T	T	T	F	F	**F**	**F**
T	F	T	F	T	**F**	**F**
F	T	T	T	F	**F**	**F**
F	F	F	T	T	**T**	**T**

Table 9: Truth table for $\sim (P \vee Q) \equiv (\sim P) \wedge (\sim Q)$.

Practice

What is the negation of the following statement (given we are working with the set of integers, $\mathbb{Z}$): x is an odd integer, and y is an odd integer.

Exercises

Note: These exercises will be on problem set 2.

Set Theory

1. Show that $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

2. Show that $A - (B \cap C) = (A - B) \cup (A - C)$.

Logic

3. State the negation of the following statements:

   (a) $\sqrt{3}$ is a rational number.

   (b) x is an even integer, or y is an odd integer.

4. Complete the following truth table:

P	Q	$\sim Q$	$P \wedge (\sim Q)$
T	T		
T	F		
F	T		
F	F		

5. Show that $\sim (P \wedge Q) \equiv (\sim P) \vee (\sim Q)$ are logically equivalent.