ASSIGNMENT 2 SELECTED SOLUTIONS

① $\subseteq$ LET $x \in \overline{A \cap B}$

$\Rightarrow x \notin A \cap B$

$\Rightarrow x \notin A$ OR $x \notin B$

$\Rightarrow x \in \overline{A} \cup \overline{B}$

$\supseteq$ LET $x \in \overline{A} \cup \overline{B}$

$\Rightarrow x \in \overline{A}$ OR $x \in \overline{B}$

$\Rightarrow x \notin A$ OR $x \notin B$

$\Rightarrow x \notin A \cap B$

$\Rightarrow x \in \overline{A \cap B}$

$\Rightarrow \overline{A \cap B} = \overline{A} \cup \overline{B}$

⑦ PROOF BY CONTRAPOSITIVE

LET $a < 3m+1$ AND $b < 2m+1$ AND SHOW $2a+3b < 12m+1$

SINCE $a, b, m \in \mathbb{Z}$

$\Rightarrow a \leq 3m$ AND $b \leq 2m$

$\Rightarrow 2(a) + 3(b) = 6m + 6m$

$= 12m < 12m+1$

(11) BASE CASE:

$n = 1$

$1 \leq 2 - 1/2 = 1.5$ ✓

ASSUME $P(k)$:

$1 + \frac{1}{4} + \frac{1}{9} + \ldots + \frac{1}{k^2} \leq 2 - \frac{1}{k}$

NEED TO SHOW:

$1 + \frac{1}{4} + \frac{1}{9} + \ldots + \frac{1}{k^2} + \frac{1}{(k+1)^2} \leq 2 - \frac{1}{k+1}$

$P(k) \Rightarrow$

$$1 + \frac{1}{4} + \frac{1}{9} + \ldots + \frac{1}{k^2} + \frac{1}{(k+1)^2} \leq 2 - \frac{1}{k} + \frac{1}{(k+1)^2}$$

$$= 2 - \left(1/k - \frac{1}{(k+1)^2}\right)$$

$$< 2 - \left(\frac{1}{k} - \frac{1}{k(k+1)}\right)$$

$$= 2 - \frac{1}{k}\left(1 - \frac{1}{k+1}\right)$$

$$= 2 - \frac{1}{k}\left(\frac{k+1}{k+1} - \frac{1}{k+1}\right)$$

$$= 2 - \frac{1}{k}\left(\frac{k}{k+1}\right)$$

$$= 2 - \frac{1}{k+1}$$ ✓

THUS $P(k) \Rightarrow P(k+1)$

(14) a) ASSUME $x_2 > x_1$

NEED TO SHOW $f(x_2) > f(x_1)$ or $f(x_2) - f(x_1) > 0$

$$f(x_2) - f(x_1) = e^{x_2} + 2x_2 - e^{x_1} - 2x_1$$

$$e^{x_2} - e^{x_1} + 2(x_2 - x_1)$$

NOTICE SINCE $x_2 > x_1$

$$e^{x_2} - e^{x} > 0 \text{ AND } 2(x_2 - x_1) > 0$$

THUS f IS STRICTLY INCREASING

(17) RECALL A SEQUENCE $\{x_n\}$ IS CAUCHY IFF $\forall \varepsilon > 0 \ \exists N \in \mathbb{N}$ s.t. $m, n \geq N \Rightarrow$

$$|x_m - x_n| < \varepsilon$$

LET $\varepsilon = 1$

THUS $|x_p| < |x_N| + 1 \quad \forall p > N$

LET $M = \max\{|x_1|, |x_2|, \ldots |x_{N-1}|, |x_N| + 1\}$

THUS $\{x_n\}$ IS BOUNDED.

(18) LET $\lim x_n = \lim z_n = l$

AND $x_n \leq y_n \leq z_n \quad \forall n \in \mathbb{N}$

LET $\varepsilon > 0$. WE MUST SHOW $\exists N \in \mathbb{N}$

s.t. $n \geq N$ IMPLIES $|y_n - l| < \varepsilon$

NOTICE THAT $\lim x_n = l \Rightarrow \exists N_1 \in \mathbb{N}$

s.t. $n \geq N_1 \Rightarrow |x_n - l| < \varepsilon$, OR $x_n \in (l-\varepsilon, l+\varepsilon)$

AND $\lim z_n = l \Rightarrow \exists N_2 \in \mathbb{N}$

s.t. $n \geq N_2 \Rightarrow |z_n - l| < \varepsilon$ OR $z_n \in (l-\varepsilon, l+\varepsilon)$

LET $N = \max\{N_1, N_2\}$

$\Rightarrow y_n \in (l-\varepsilon, l+\varepsilon)$, OR IN OTHER

WORDS $|y_n - l| < \varepsilon$ FOR $\forall n \geq N$