# Assignment 2

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## Set Theory

List out all the elements of each set, and put the elements within curly brackets { and }.

1. $A = \{n \in \mathbb{N} : n < 10\}$
2. $B = \{n \in \mathbb{Z} : x^2 < 6\}$
3. $C = \{x \in \mathbb{R} : x^2 + 1 = 0\}$

Put the following sets in set builder notation. In other words, write each set in the form $\{x \in \mathbb{Z} : p(x)\}$, where $p(x)$ is an expression of $x$.

5. $D = \{-1, -2, -3, \ldots\}$
6. $E = \{-9, -4, -1, 0, 1, 4, 9\}$
7. $F = \{-1, 0, 1, 8, 27\}$

Let $U$ be a universal set, and let $A$ and $B$ be subsets of $U$. Draw a venn diagram for the following sets:

8. $\overline{A \cup B}$
9. $\overline{A} \cup \overline{B}$

10. Let $U = \{1, 2, 3, \ldots, 15\}$ be the universal set, $A = \{1, 3, 8, 9\}$, and $B = \{2, 8, 15\}$. Determine the following:
    (a) $A \cup B$
    (b) $A \cap B$
    (c) $A - B$
    (d) $\overline{A}$
    (e) $A \cap \overline{B}$

## Direct Proof

11. Let $n \in \mathbb{Z}$. Prove that if $n$ is even, then $3x - 11$ is odd.
12. Let $x, y, z \in \mathbb{Z}$. Prove that if $x$ and $z$ are odd, then $xy + yz$ is even.

## Proof by Contrapositive

13. Let $n \in \mathbb{Z}$. Prove that if $9x + 3$ is odd, then $x$ is even.

## Proof by Contradiction

14. Prove that is $x$ and $y$ are positive real numbers, then $\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$

## Proof by Cases

15. Let $a, b \in \mathbb{Z}$. Prove that $a - b$ is even if and only if $a$ and $b$ are of the same parity (either both are even or both are odd).

## Mathematical Induction

16. Prove that

$$1 + 5 + 9 + 13 + \ldots + (4n - 3) = 2n^2 - n$$

for every $n \in \mathbb{N}$

## Relations and Functions

17. Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by $f(x) = x^2 + ax + b$ where $a, b \in \mathbb{R}$. Show that $f$ is not one-to-one.

## Set Theory Proofs

De Morgan's Laws are defined as such:

$$\overline{A \cup B} = \overline{A} \cap \overline{B} \tag{1}$$
$$\overline{A \cap B} = \overline{A} \cup \overline{B} \tag{2}$$

In order to show that two sets, $X$ and $Y$, are equal, we need to show that every element of $X$ is an $Y$, and every element of $Y$ is in $X$. In other words, we need to show that $X \subseteq Y$ and $Y \subseteq X$. Suppose I wanted to show that the first expression was true, in other words that $\overline{A \cup B} = \overline{A} \cap \overline{B}$. To show the equality holds, I need to show that $\overline{A \cup B} \subseteq \overline{A} \cap \overline{B}$ and $\overline{A} \cap \overline{B} \subseteq \overline{A \cup B}$. I'll first show that $\overline{A \cup B} \subseteq \overline{A} \cap \overline{B}$.

**Proof** Assume that $x \in \overline{A \cup B}$.
$\Rightarrow x \notin (A \cup B)$
$\Rightarrow x \notin A$ and $x \notin B$
$\Rightarrow x \in \overline{A}$ and $x \in \overline{B}$
$\Rightarrow x \in \overline{A} \cap \overline{B}$

Next, in order to show that $\overline{A} \cap \overline{B} \subseteq \overline{A \cup B}$, we would assume an arbitrary $x \in \overline{A} \cap \overline{B}$, and show that $x \in \overline{A \cup B}$. Although essential to the proof, I have left this part out.

18. Prove that expression (2) holds. Namely, that $\overline{A \cap B} = \overline{A} \cup \overline{B}$. Use the technique outlined above.

19. Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ using the technique outlined above.